be $\frac{13-5c^2}{8c}$. Hence $26c-10c^3$ must be a square; it is evident that this is the case when $c=1$. Then $b=1$ and $a=\frac{9}{4}$. Substituting these in the values of m and n, and we have $m=\frac{41}{9}$ and $n=\frac{185}{153}$. Taking $x=153$, we have $mx=697$, and $nx=185$, and the numbers are $(153)^2$, $(185)^2$, and $(697)^2$.

Also solved by *G. B. M. ZERR*, and *EDWARD D. GRABER*.

AVERAGE AND PROBABILITY.

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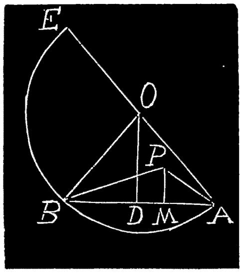
111. Proposed by LON C. WALKER, A.M., Professor of Mathematics, Petaluma High School, Petaluma, Cal.

If a radius be drawn at random in a given semi-circle, and a point taken at random in one of the sectors formed, show that the chance that a random line drawn through the point will cut the arc of the sector is $1-\frac{1}{\pi^2}\log 2$.

Solution by the PROPOSER.

Let ABE be the given semicircle, OB the random radius, P the random point, OD and PM perpendicular to AB.



Put $DM=x$, $PM=y$, $OA=1$, $\angle AOB=\theta$, $\angle APM=\phi$, $\angle BPM=\psi$. Then $AD=\sin\frac{1}{2}\theta$, $OD=\cos\frac{1}{2}\theta$, area of segment $ACB=\frac{1}{2}(\theta-\sin\theta)$, $\phi=\tan^{-1}\left(\frac{\sin\frac{1}{2}\theta-x}{y}\right)$, $\psi=\tan^{-1}\left(\frac{\sin\frac{1}{2}\theta+x}{y}\right)$

When P is in the segment ACB the random line will cut the arc whatever be its direction, and when P is in the triangle AOB the number of favorable directions of the random line will be $2(\phi+\psi)$. Hence we have

$$p=\frac{\int_0^\pi \pi(\theta-\sin\theta)d\theta+\int_0^\pi\int_0^{\cos\frac{1}{2}\theta}\int_{-\tan\frac{1}{2}\theta(\cos\frac{1}{2}\theta-y)}^{\tan\frac{1}{2}\theta(\cos\frac{1}{2}\theta-y)} 2(\phi+\psi)dxdyd\theta}{\int_0^\pi \pi\theta d\theta}$$

$$=1-\frac{4}{\pi^2}+\frac{4}{\pi^3}\int_0^\pi\int_0^{\cos\frac{1}{2}\theta}\Big[2\tan\tfrac{1}{2}\theta(2\cos\tfrac{1}{2}\theta-y)\tan^{-1}\tan\tfrac{1}{2}\theta\left(\frac{2\cos\frac{1}{2}\theta-y}{y}\right)$$

$$-y\theta\tan\tfrac{1}{2}\theta-y\log\left(\frac{y^2+\tan^2\frac{1}{2}\theta(2\cos\frac{1}{2}\theta-y)^2}{2y^2}\right)\Big]dy\,d\theta$$

$$=1-\frac{4}{\pi^2}+\frac{4}{\pi^3}\int_0^\pi\Big[\theta\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta-2(\theta-\pi)\sin^3\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta-2\sin^2\tfrac{1}{2}\theta\cos^2\tfrac{1}{2}\theta\log^2$$

$$+\tfrac{1}{4}\log\left(\frac{1+\cos\theta}{1-\cos\theta}\right)+\tfrac{1}{4}\cos\theta\log(1+\cos\theta)+\tfrac{1}{2}\cos2\theta\log(1-\cos\theta)\Big]d\theta=1-\frac{1}{\pi^2}\log 2.$$

Solved with same result by *F. P. MATZ*. Professor Zerr gets as a result $1-(1/4\pi^2)(8\log 2+7)$.